



Analysis of mortality curves: non parametric density estimation using bayesian approach

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The model

This work has the aim to propose a flexible model for mortality curves by age based on bayesian nonparametrics.

Let $x_i, i = 1, \dots, n$, be the **unknown** exact age at death of i -th subject. Then

$$x_i | \tilde{p} \stackrel{\text{iid}}{\sim} \tilde{p}$$
$$\tilde{p} | \theta \sim \text{DP}(\alpha, P_0(\cdot; \theta)) .$$

The fundamental idea is to consider the **known** deaths distribution by age classes as a realization from a random variable with multinomial distribution:

$$d_j = \sum_{i=1}^n \mathbb{I}(x_i \in [i, i + 1)), \quad j = 0, \dots, 110;$$

$$d_0, \dots, d_{110} | \tilde{p} \sim \text{Multinom}(n, \pi)$$

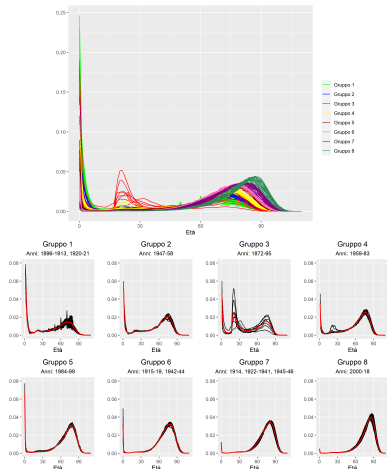
The model

This allows (Gelman et al., 2013) to set as prior distribution for the unknown parameter π a probability measure induced by a Dirichlet process with base measure P_0 chosen following Aliverti et al. (2021).

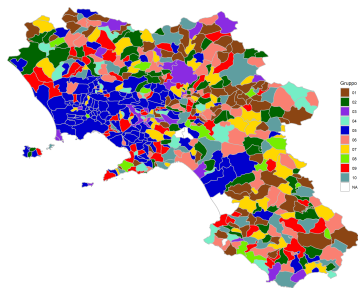
Furthermore, considering multiple curves, an extension of this model provides a model-based clustering procedure which can identify and group together curves with same features.

The parameters estimation is made by a three-step MCMC algorithm.

Results of two analysis








(a) *Situation 1: analysis of Italian male population curves since 1872.*



(b) *Situation 2: analysis of Italian male population curves by county in 2020, results for Campania region.*

References

-  Aliverti, E., Mazzuco, S. e Scarpa, B. (2021). Dynamic modeling of mortality via mixtures of skewed distribution functions. arXiv preprint arXiv:2102.01599.
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