



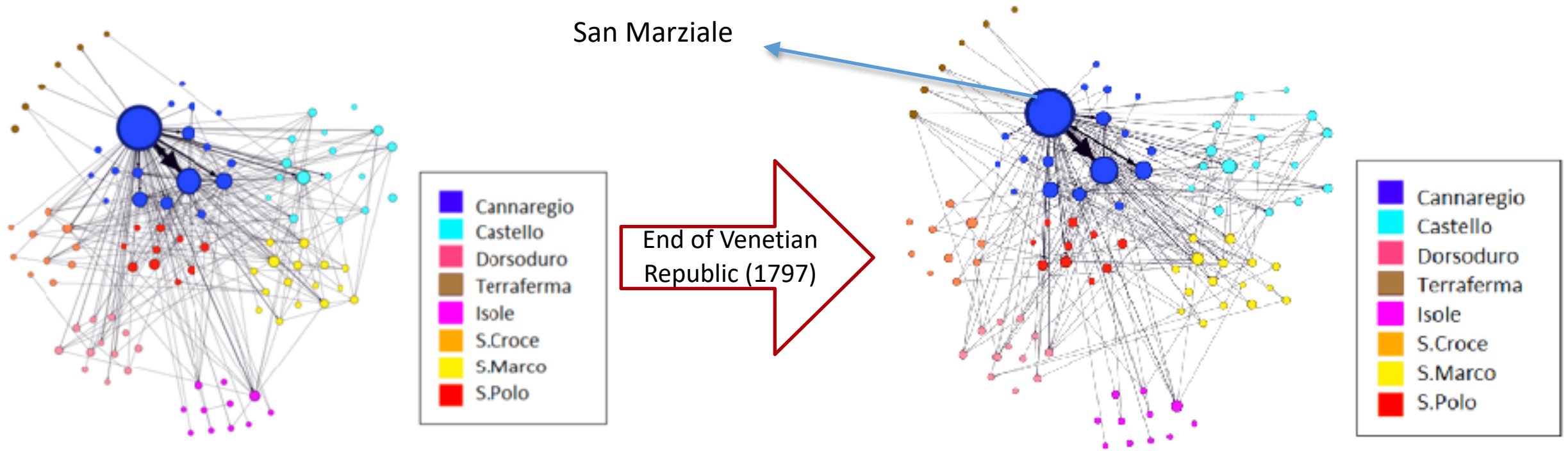
## **Comparison of Venice friendship network between 18th and 19th century: a marital acts analysis.**

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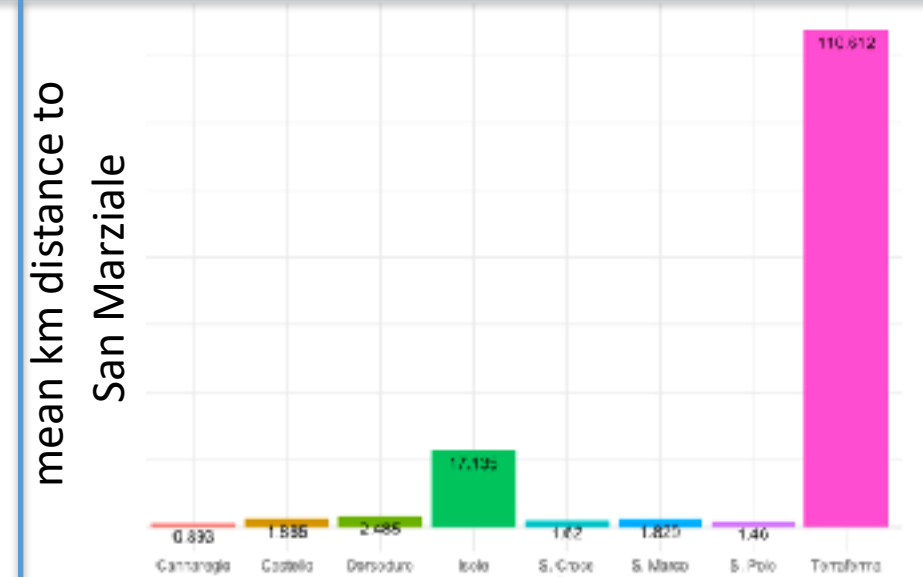
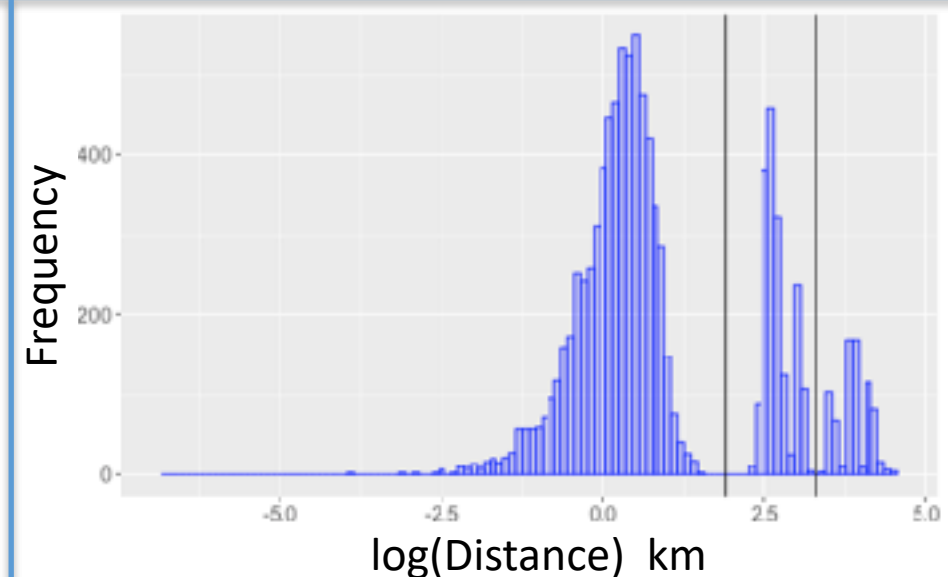
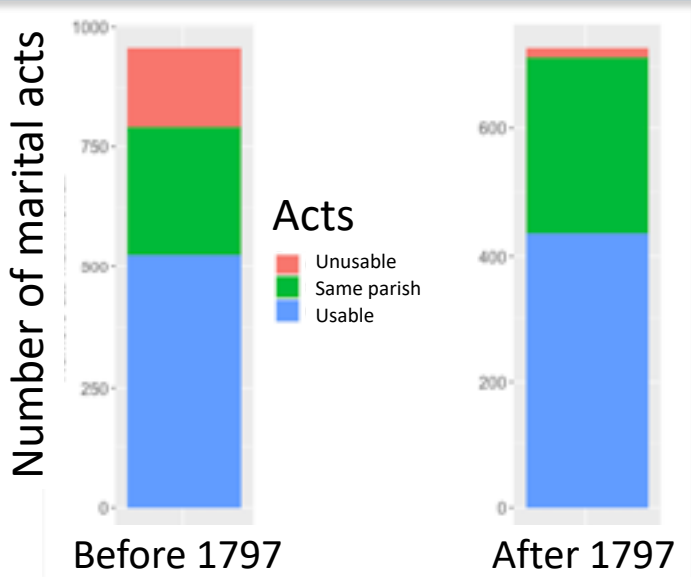
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Graphs of friendship bonds; colors represent areas, edges thickness are proportional to the weight and nodes size to the strength.



## Rounded Normal (NR) network model

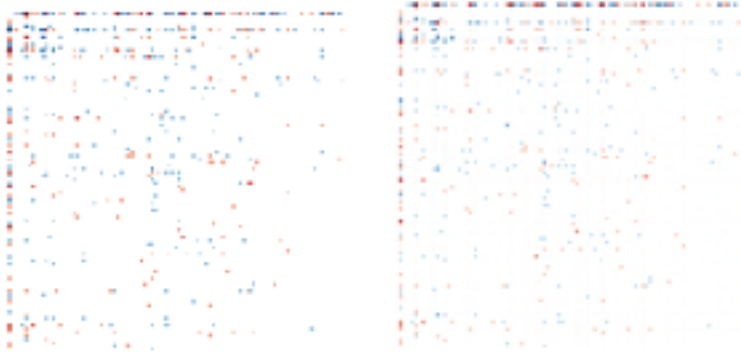
$$\eta_{i,j} = \mu + \beta^T x_{i,j} + a_i + b_j + u_i^T v_j$$

$$z_{i,j} \sim N(\eta_{i,j}, \sigma^2)$$

$$y_{i,j} | z_{i,j} = I(k - 1 < z_{i,j} \leq k)$$

$$\text{pr}(y_{i,j} = k | z_{i,j}) = \text{pr}(k - 1 < z_{i,j} \leq k) \quad k = -\infty, \dots, 0, 1, \dots, \infty$$

$$\text{pr}(y_{i,j} = k) = \int_{k-1}^k f(z_{i,j}) dz_{i,j} = \Phi(k; \eta_{i,j}, \sigma^2) - \Phi(k-1; \eta_{i,j}, \sigma^2).$$



Differences of weight matrix: observed (left) and fitted (right). Red if there is a decrease in citations, blue viceversa.

$$\{(a_1, b_1), \dots, (a_n, b_n)\} \sim N_2(0, \Sigma_{ab})$$

$$\{(u_1, v_1), \dots, (u_n, v_n)\} \sim N_{2n}(0, \Psi)$$

$$\{(\epsilon_{ij}, \epsilon_{ji})\} \sim N_2(0, \Sigma_\epsilon)$$

$$\beta \sim N_p(\mu_\beta, \Sigma_\beta)$$

$$\Sigma_{ab} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix}$$

$$\Sigma_\epsilon = \sigma_\epsilon^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

## overdispersed Poisson network model

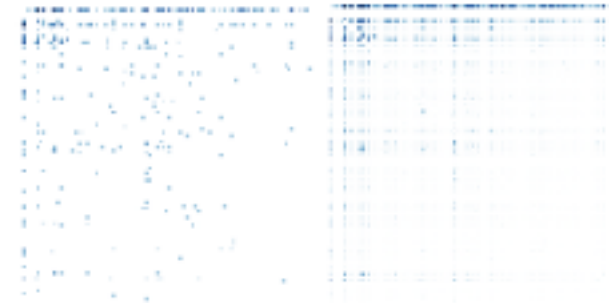
$$z_{ij} = \mu + a_i + b_j + \beta x_{ij} + u_i^T v_j + \epsilon_{ij}$$

$$y_{ij} | z_{ij} \sim \text{Poisson}(e^{z_{ij}})$$

the global parameters  $\beta, \Sigma_{ab}, \Sigma_{uv}, \sigma^2, \rho$ ;

the latent nodal effects  $\{(a_i, b_i, u_i, v_i) : i = 1, \dots, n\}$

the unobserved latent dyadic variables  $z_{i,j}$ .

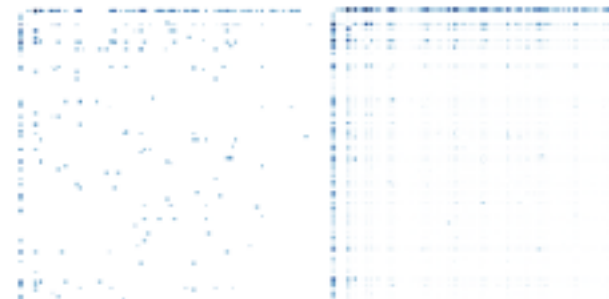


Weight matrix observed (left) and fitted (right), before 1797 (above) and after 1797 (below)

$$\beta_{Dist} = -7,76$$

$$\sigma^2 = 0,984$$

Denotes overdispersion



$$\beta_{Dist} = -4,52$$

$$\sigma^2 = 0,942$$

Denotes overdispersion